PERFORMANCE ANALYSIS OF RADIO OVER FIBER SYSTEM WITH EXTERNAL MODULATION INCLUDING THE EFFECT OF FIBER DISPERSION

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Abstract:
Radio-over fiber technology (RoF) fulfills the demand of future wireless services. In this paper, simulative analysis of radio-over fiber system employing external modulation using Mach-Zehnder modulator is carried out by considering the effect of fiber dispersion. Analytical and simulative analysis shows the periodic behaviour of system with respect to modulation index and length of the optical fiber. The present analysis shows that the fading of radio frequency (RF) signal amplitude having cyclic variation depends on modulation index. In this paper the fundamental and upto 2nd order harmonics of modulation sidebands which are generated by nonlinear response of optical modulator are studied.

Keywords: Electro-optic modulation, optical fiber dispersion, optical propagation in dispersive media, Mach-Zehnder modulator (MZM).

I. Introduction
Growing demands for high speed mobile communications are encouraging the deployment of novel technologies which associates the wired and wireless services. Radio over fiber transmission is one of the key techniques for fulfilling such demands in future mobile communications [1]. Radio-over-fiber (RoF) refers to a technique where light is modulated by a radio signal and transmitted over an optical link to extend the wireless access. It is basically an integration of RF and optical network which have higher bandwidth, immunity to RF interference and having less cost and less power consumption. At a central station, the RF signals are used to directly modulate the intensity of laser light or an external optical modulator can be used to modulate the intensity of the laser light, which is then transmitted through an optical fiber. At each base station, a photo detector converts the received optical signals back to the original RF signals.

It is well known that the RoF transmission of a double-sideband (DSB) optical signal experiences a dispersion induced signal fading [2]. Dispersion is the time domain spreading or broadening of the transmission signal light pulses as they travel along the fiber. It is a phenomenon that deals with the fact that the velocity of the electromagnetic wave depends on the wavelength. When an optical carrier is simply intensity modulated, the optical field comprises the carrier and the two optical sidebands. Now dispersion causes a phase difference between these modulation sidebands. If the relative phase difference becomes \(\pi\), then the contribution of the sidebands to the change in optical amplitude becomes destructive. Here amplitude modulation is converted to phase modulation. This problem can be resolved by eliminating one of the sideband i.e. by using optical single-sideband transmission.

For improving spectral efficiency, complex multilevel modulation formats such as quadrature-amplitude modulation are used for radio services. In QAM, phase and amplitude are modulated simultaneously and signals having multiple amplitude levels are produced. In this paper, we analyze the effect of fiber dispersion on the RoF transmission both analytically and numerically by considering the effect of modulator and detector nonlinearities which are given as:

a) Intensity modulation and amplitude modulation are not equivalents of each other. Their equivalence is possible only when modulation depth is moderate.

b) The nonlinearity at the detector level is due to the direct detection process of an optical signal power not the amplitude.
Here we consider two important topics:
1) The amplitude ratio between the carrier and the fundamental modulation sidebands is a function of modulation depth.
2) When the modulated signal is detected by a photodetector, beat signals between the higher order harmonics which are generated by the intensity modulator, fall in the frequency range of the proper signal [5].

II. Dispersion effect analysis using Mach-Zehnder Modulator

Consider a sinusoidal RF wave with an angular frequency \( \omega_1 \) represented as
\[
x(t) = A \sin(\omega_1 t)
\]

By considering the intensity modulation of this RF wave, we write an initial light field in a Fourier series as
\[
e(0, T) = \sqrt{U_0} \sum_{k=0}^{\infty} C_k(m) \cos k\omega_1 T
\]
where each coefficient \( C_k(m) \) corresponds to the optical amplitude of the kth harmonic modulation sideband and is a function of the modulation depth \( m \) that lies in the range \((0, 1)\) and \( U_0 \) is a parameter proportional to the average power.

The Fourier Transform of (1) can be represented as:
\[
E(0, \omega) = 2\pi \sqrt{B_0} \sum_{k=0}^{\infty} C_k(m) \frac{\delta(\omega-k\omega_1)+\delta(\omega+k\omega_1)}{2}
\]
where \( \delta(x) \) is Dirac’s delta function.

To get an expression for the light field at a particular propagation distance i.e. \( L \), we use the standard equation of optical field in purely dispersive fiber represented as
\[
\frac{\partial e(L,T)}{\partial L} = i\beta_2 \frac{\partial^2 e(L,T)}{\partial T^2}
\]
where \( T \) is the time scale (the origin of which moves along with the optical signal) and \( \beta_2 \) is the second order dispersion of the transmission fiber.

The Fourier Transform pair for \( e(L,T) \), defined by
\[
E(L, \omega) = \int_{-\infty}^{\infty} e(L, T) \exp(-i \omega T) dT
\]
\[
e(L, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{E(L, \omega) \exp(i \omega T) d\omega}{\omega}
\]

By using (3), (4), (5) we get the corresponding equation for Fourier Transform of optical field in purely dispersive fiber:

\[
\frac{dE(L,\omega)}{dL} = -i \frac{\beta_2 \omega^2}{2} E(L, \omega)
\]
whose analytical solution given by:
\[
E(L, \omega) = E(0, \omega) \exp\left(-i \frac{\beta_2 \omega^2 L}{2}\right)
\]

By substituting the value of \( E(0, \omega) \) from (2) into (7) and calculating its inverse Fourier transform, we get
\[
e(L, T) = \sqrt{U_0} \sum_{k=0}^{\infty} C_k(m) \cos k\omega_1 T \exp(-ik^2 \theta)
\]
where
\[
\theta = \frac{\beta_2 \omega_1^2 L}{2}
\]
is the fading parameter.

As we know that the output current of a photodetector is proportional to \( |e(L, T)|^2 \), the transmitted RF signals can be evaluated using \( |e(L, T)|^2 \) normalized by the power parameter \( U_0 \)
\[
\frac{|e(L, T)|^2}{U_0} = C_0^2(m) + \sum_{k=1}^{\infty} C_k^2(m) \cos^2 k\omega_1 T
\]
\[
+ 2 \sum_{k=1, k+1 \leq p} C_k(m) C_p(m) \times \cos k\omega_1 T \cos \omega_1 T \cos(k^2 - p^2) \theta
\]
\[
\equiv \sum_{n=0}^{\infty} s_n(m, \theta) \cos n\omega_1 T
\]
where \( s_n(m, \theta) \) is used to define the amplitude of the nth harmonic of the transmitted RF signal.

ROF transmission considers only the fundamental signal (n=1), the normalized amplitude of which can be expressed as
\[
s_1(m, \theta) = 2C_0(m)C_1(m) \cos \theta + \sum_{k=1}^{\infty} C_k(m) C_{k+1}(m) \cos(2k + 1) \theta
\]

If we consider the effect of harmonic sidebands up to the second order in (10) due to their dominance, \( s_1(m, \theta) \) is approximated as
\[
s_1(m, \theta) \approx 2C_0(m)C_1(m) \cos \theta
\]
\[
\times \left[ 1 + \frac{C_2(m) \cos 3\theta}{C_0(m) \cos \theta} \right]
\]
\[
s_1(m, \theta) = 2C_0(m)C_1(m) \cos \theta
\]
\[
\times \left[ 1 + \frac{C_2(m) (2\cos 2\theta - 1)}{C_0(m) \cos \theta} \right]
\]

Here (11) suggests that the signal amplitude varies according to \( 2\theta \). This \( 2\theta \) dependence originates from the beat signal between the
fundamental and second harmonic sidebands. But if we neglect the second term in the square brackets i.e. the component due to propagation of second order and higher order harmonic sidebands, we obtain a conventional expression of dispersion induced fading that varies with the parameter $\theta$. This can be numerically confirmed in Section III.

However we should note that $s_1(m, \theta)$ depends on the value of $m$ (modulation depth). $s_1(m, \theta)$ is proportional to $m$ if $m$ is small and if $m$ is large then $s_1(m, \theta)$ varies nonlinearly with $m$.

There is a one of the possibilities of external modulation is to use Mach-Zehnder intensity modulator which will clearly explain these issues. Mach-Zehnder intensity modulator (MZM) provides both the required bandwidth and important means for minimizing the effects of dispersion. There are two basic types of configurations of MZM. One is push pull configuration in which applying data and bias voltage in one arm and inverted data and inverted bias voltage in other arm is applied. Other is asymmetric configuration where the modulating signal and the bias voltage are applied to one of the arms, either to the same or to different arms. In this paper push pull configuration is used [3].

Here the incoming light is split into two arms under the influence of conducting electrodes. The electro-optical effect induces a change in the refractive index of the each arm due to voltage applied to the arms. This changing refractive index phase modulates the beam propagating through the arms. By combining two paths, this phase modulation gets converted to intensity modulation.

![Figure 1: Block Diagram of RoF transmission using MZ modulator](image)

The initial output field for MZ intensity modulator i.e. for $L=0$ can be expressed as

$$e(0, T) = \frac{u_0}{2} \cos \frac{\pi V}{2V_r} \cos \frac{\pi}{4}(1 + m \cos \omega_1 T)$$

where $V$ is the applied voltage given by $V = V_B + V_m \cos \omega_1 t$.

Here $V_B$ is the bias voltage, $V_m$ is the amplitude of electrical RF signal and $V_r$ is the half-wavelength voltage. If we bias the modulator by $V_r/2$ then we get

$$e(0, T) = \frac{u_0}{2} \left\{ \cos \left( \frac{\pi}{4}(1 + m \cos \omega_1 T) \right) \right\}$$

By using these algebraic relationships

$$\cos(y \cos \omega_1 T) = J_0(y) - 2J_2(y) \cos 2\omega_1 T + 2J_4(y) \cos 4\omega_1 T + \cdots$$

$$\sin(y \cos \omega_1 T) = 2J_1(y) \cos \omega_1 T - \cdots$$

The Fourier series for (14) can be given as

$$e(0,T) = \frac{1}{\sqrt{V_0}} \left[ J_0(\pi m/4) - J_1(\pi m/4) \cos \omega_1 T - \frac{J_2(\pi m/4) \cos 2\omega_1 T \cdots}{2} \right]$$

where $J_n(y)$ is the $n$th-order Bessel function of the first kind. The coefficients of the first three terms of (15) are:

$$C_0(m) = \frac{1}{2}J_0(\pi m/4)$$

$$C_1(m) = -J_1(\pi m/4)$$

$$C_2(m) = -J_2(\pi m/4)$$
### III. Result and Discussion

Numerical simulation has been done for relative harmonic amplitude of the transmitted RF signal using MATLAB. Its variation has been studied with the change in fiber length for different radio frequency and modulation index by using Mach-Zehnder intensity modulator.

Fig. 2(a) shows the graph between relative fundamental amplitude of the transmitted RF signal and the length of the optical fiber at a particular radio frequency of 10 GHz for the three different modulation indices i.e. for m=0.3, m=0.5, m=0.7. From the figure it can be shown that with increase in modulation depth, relative fundamental amplitude increases with increase in fiber length. Here the fiber length varies from 0 to 30 km.

Variation of relative fundamental amplitude with length for each modulation depth is sinusoidal due to its dependence on fading factor which is a function of fiber length.
Figure 2 (b): Relative Harmonic Amplitude Vs Length at radio frequency 10GHz

of 10 GHz for the three different modulation indices has been shown in Fig.2(b). Here the relative harmonic amplitude of the transmitted RF signal includes fundamental and the harmonic sidebands up to the second order. In this the optical fiber length is varied from 0 to 30 km. For m=0.3, relative harmonic amplitude of RF signal oscillates between -0.12 & +0.12 whereas for m=0.5 it fluctuates between -0.175 and +0.175. Similarly for m=0.7 it varies from -0.225 to +0.225.

Fig.2(c) shows the impact of angular frequency on the variation for relative harmonic amplitude of the transmitted RF signal with respect to the optical fiber length which varies from 0 to 30 km, at a particular modulation index i.e. for m=0.5. Here variation is studied with respect to two angular frequencies of the transmitted RF signal, one is 10GHz and other is 20 GHz. Relative harmonic amplitude depends on cos θ where θ is the fading parameter which is directly proportional to the angular frequency of the transmitted RF signal i.e. ω. Therefore it shows sinusoidal variation with ω. Here the RF amplitude fluctuates in a different manner but within the same range i.e. from -0.175 to +0.175 for both the radio frequencies.

Figure 2 (c): Relative Harmonic Amplitude Vs Length for m=0.5

IV. Conclusion
Numerical and simulative analysis has been done to study the effect of fiber dispersion on fundamental and harmonics of modulation sidebands. It is found that the RF amplitude faded in cyclic manner with respect to modulation index and length of optical fiber at various RF frequencies.

V. References
